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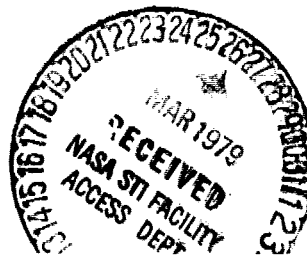
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NRLO

**(NASA-Case-NPO-14525-1) MULTIBEAM SINGLE  
FREQUENCY SYNTHETIC APERTURE RADAR PROCESSOR  
FOR IMAGING SEPARATE RANGE SWATHS Patent  
Application (NASA) 16 p HC A02/NF A01**

**N79-19195****Unclass****CSCL 17I G3/32 43756**

# AWARDS ABSTRACT

Inventor: Atul Jain

~~JPL Case No. 14525~~

NASA Case No. NPO-14525-

Contractor: Jet Propulsion Laboratory

~~January 17, 1979~~

## MULTIBEAM SINGLE FREQUENCY SYNTHETIC APERTURE RADAR PROCESSOR FOR IMAGING SEPARATE RANGE SWATHS

The invention described herein was made in the performance of work under a NASA contract and is subject to the provisions of Section 305 of the National Aeronautics and Space Act of 1958, Public Law 85-568 (72 Stat. 435; U.S.C. 2457).

This invention relates to synthetic aperture radar (SAR), and more particularly to a method and apparatus for single frequency multibeam imaging of multiple strips of range swath at high range intervals for those applications where it is desirable to cover a range swath much greater than is possible for a given interpulse interval.

FIG. 1 illustrates the method and FIGs 2a and 2b illustrate two alternative arrangements of apparatus for carrying out the method using a tilted plane processor. A single frequency multibeam synthetic aperture radar system is achieved by a data processing method which separates images in the radar data for different beams on the basis of the Doppler frequency spectrum of the beams. A single frequency synthetic aperture radar is employed to develop the data to be processed over a total range and azimuth which will encompass both the range swaths and azimuth of the multiple beams. The beam parameters are selected so that the return from each successive range swath is received during successive interpulse periods of the radar system, and so that the return from each beam may be separated on the basis of its Doppler frequency spectrum at the frequency plane of the processor. The processing method comprises selecting the multiple beams at different ranges on the basis of their Doppler frequency spectrum. This may be accomplished by selective narrow band filtering the frequency spectrum of the radar data while recording the image for each successive range swath, or spatially separating the image data of all beams simultaneously, and selecting the image for each successive range swath in succession. An exemplary technique is to use an optical system to process the radar data recorded on film as shown in FIG. 2a. The optical system consists of a tilted plane correlator having a Fourier transform lens  $L_1$ , a frequency plane aperture filter  $F$ , an imaging lens  $L_2$  to convert the range data into a range image, and a cylindrical azimuth telescope  $L_3$ . By controlling the size and position of frequency plane aperture filter, one beam may be selected for recording at a time. Alternatively, by using a large and stationary frequency plane filter to pass all frequency spectrums as shown in FIG. 2b, the azimuth telescope separates the image data for each beam so that by controlling the size and position of a slit directly in front of a recording film, the image for each successive range swath may be selected for recording. All of the functions of this optical system may be implemented using electrical circuits in strictly analogous ways.

The novelty of the invention resides in using the Doppler history and the interpulse periods of data return to separate the SAR data into multiple beams for imaging a plurality of range swaths, where the ranges of the swaths are greater than the interpulse periods of the SAR for all but possibly the first swath.

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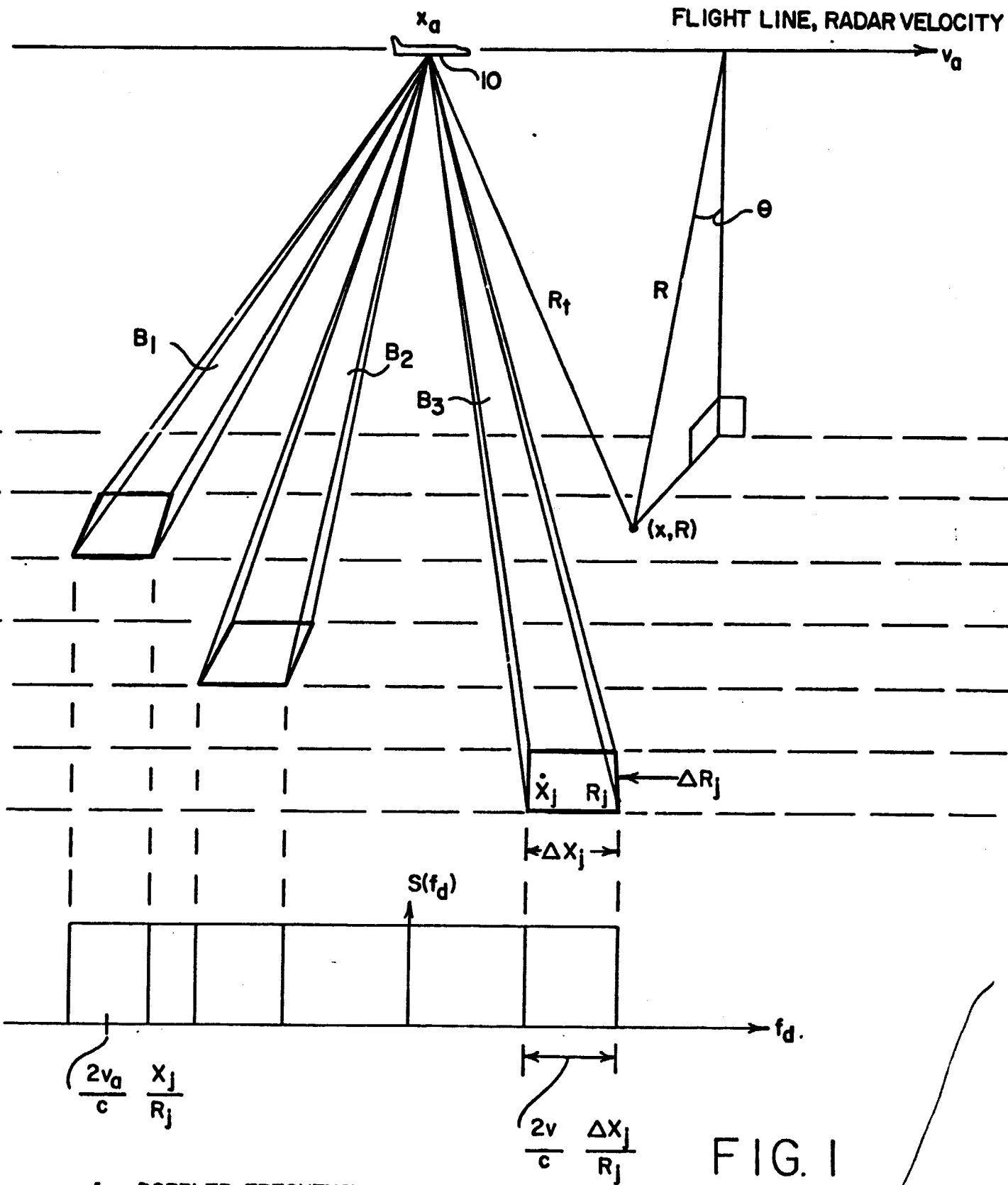


FIG. 1

$f_d$  = DOPPLER FREQUENCY  
 $S(f_d)$  = SPECTRUM IN DOPPLER FREQUENCY

017885 25%

FIG. 2a

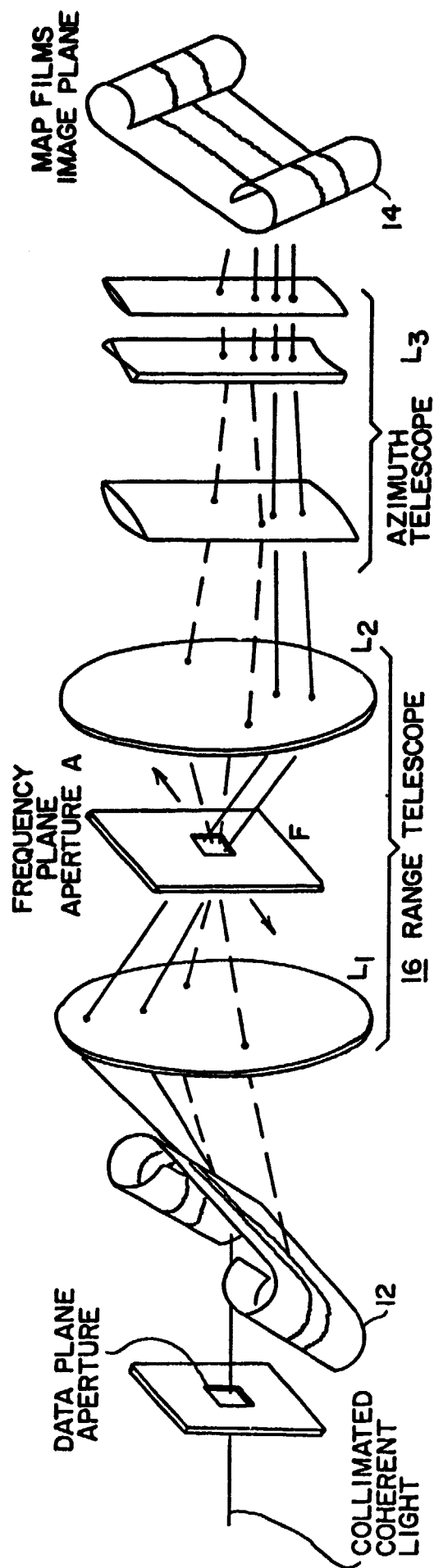
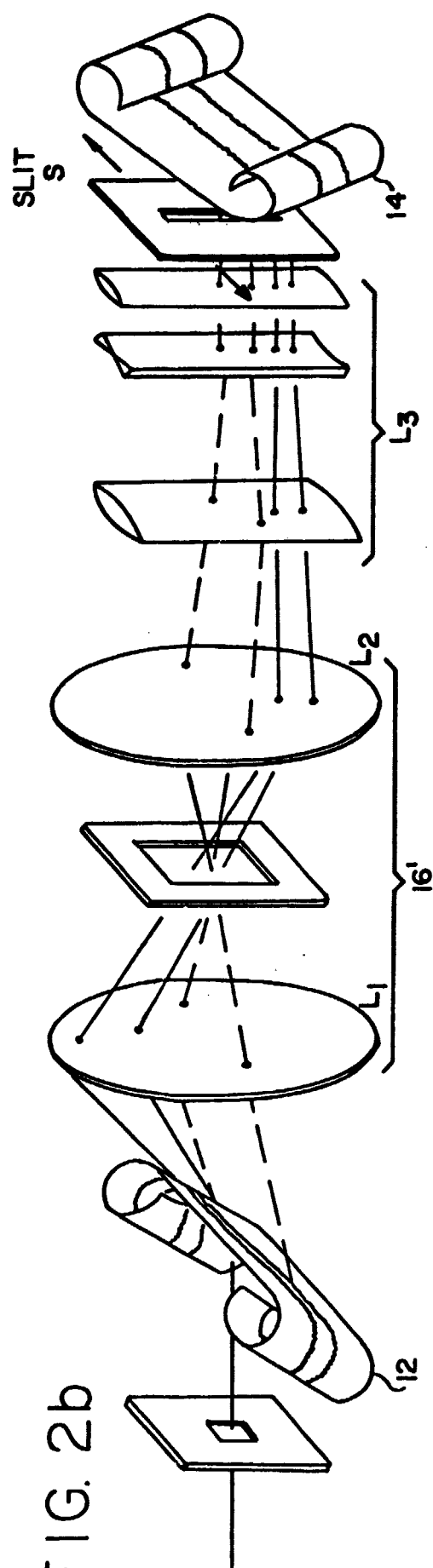


FIG. 2b



Serial No.	017885
Filing Date	3/6/79
Contract No.	NAS7-100
Contractor	Caltech/JPL
Pasadena, CA	91106
(City) (State) (Zip)	

78/189  
JPL Case No. 14525  
NASA Case NPO-14525

TO ALL WHOM IT MAY CONCERN:

BE IT KNOWN THAT I, ATUL JAIN, a citizen of India, residing at Altadena, in the County of Los Angeles, State of California, have invented a new and useful

5      **MULTIBEAM SINGLE FREQUENCY SYNTHETIC APERTURE  
RADAR PROCESSOR FOR IMAGING SEPARATE RANGE SWATHS**

**ABSTRACT**

10      Data from a single-frequency synthetic aperture  
radar (in which beam parameters are adjusted so that the  
return from each successive swath is received during suc-  
cessive interpulse periods) are separated in Doppler frequency  
for the return from each beam at the frequency plane of  
the processor. Alternatively, the image formed by each beam  
may be spatially separated in the azimuth direction and  
15      successively selected by positioning an appropriate slit in  
the recording plane of the processor.

## ORIGIN OF INVENTION

The invention described herein was made in the performance of work under a NASA contract and is subject to the provisions of Section 305 of the National Aeronautics and Space Act of 1958, Public Law 85-568 (72 Stat. 435; U.S.C. 2457).

## BACKGROUND OF THE INVENTION

This invention relates to synthetic aperture radar (SAR), and more particularly to a method and apparatus for single frequency multibeam imaging of multiple strips of range swath at high range intervals.

In some applications for SAR, it is desirable to cover a range swath much greater than is possible for a given interpulse interval. For example, in global oceanography, SAR may be used to monitor ocean states. The ocean wave spectra does not change very rapidly spatially, and so samples of the wave spectra extending over a large area are necessary to enable modeling and prediction of ocean conditions. When the SAR sends out a succession of pulses, the time delay for the pulse return provides the range resolution and Doppler processing of the returns for a series of pulses provides the azimuth resolution for the images.

While the time between successive pulses determines the maximum range that can be mapped by the radar system, too low pulse repetition frequency results in azimuth ambiguities due to Doppler foldover of the matched filtered image output. The unambiguous range that can be mapped is given by  $\frac{cD}{8v_a}$ , where  $c$  is the velocity of light,  $D$  the antenna dimension and  $v_a$  the velocity of the radar platform. This

is insufficient unambiguous range for oceanography which requires monitoring ocean conditions over a range up to 15000 km. What is proposed for this application is an SAR processor for image swaths of 10 km widths centered at 100  
5 km intervals up to the range of 1500 km. This requires a multibeam system, each beam illuminating a different range swath and pointing at a separate azimuth angle. The problem is achieving this with a single frequency radar system.

#### SUMMARY OF THE INVENTION

10 In accordance with the present invention, a single frequency multibeam synthetic aperture radar system is achieved by a data processing method which separates images in the radar data for different beams on the basis of the Doppler frequency spectrum of the beams. A single frequency syn-  
15 thetic aperture radar is employed to develop the data to be processed over a total range and azimuth which will encompass both the range swaths and azimuth of the multiple beams. The beam parameters are selected so that the return from each successive range swath is received during successive inter-  
20 pulse periods of the radar system, and so that the return from each beam may be separated on the basis of its Doppler frequency spectrum at the frequency plane of the processor. The processing method comprises selecting the multiple beams at different ranges on the basis of their Doppler frequency  
25 spectrum. This may be accomplished by selective narrow band filtering the frequency spectrum of the radar data while recording the image for each successive range swath, or spatially separating the image data of all beams simultaneously, and selecting the image for each successive range



swath in succession. An exemplary technique is to use an optical system to process the radar data recorded on film. The optical system consists of a tilted plane correlator having a Fourier transform lens  $L_1$ , a frequency plane aperture filter  $F$ , an imaging lens  $L_2$  to convert the range data  
5 into a range image, and a cylindrical azimuth telescope  $L_3$ . By controlling the size and position of frequency plane aperture filter, one beam may be selected for recording at a time. Alternatively, by using a large and stationary frequency plane  
10 filter to pass all frequency spectrums, the azimuth telescope separates the image data for each beam so that by controlling the size and position of a slit directly in front of a recording film, the image for each successive range swath may be selected for recording. All of the functions of this optical  
15 system may be implemented using electrical circuits in strictly analogous ways.

The novel features that are considered characteristic of this invention are set forth with particularity in the appended claims. The invention will best be understood  
20 from the following description when read in connection with the accompanying drawings.

#### BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 shows radar image geometry useful in understanding the present invention.

25 FIG. 2a illustrates the configuration of a tilted plane optical processor which may be used to carry out the present invention.

FIG. 2b illustrates an alternative for the processor of FIG. 2a.

### DESCRIPTION OF PREFERRED EMBODIMENTS

To implement the present invention with side-looking synthetic aperture radar (SAR) 10 shown in FIG. 1, the radar may have a chirped signal with radar frequency ranging from 1200  
5 MHz to 1210 MHz. The return is recorded on a data film 12, shown in FIG. 2a, which may then be optically processed to generate map films 14, each film having different range swath images.

The principle involved is to separate the return  
10 data recorded on the data film 12 into multiple beams, each beam pointing at a separate range and azimuth interval, as shown in FIG. 1 for beams  $B_1$ ,  $B_2$  and  $B_3$ . As the radar moves along a flight line at a known velocity,  $v_a$ , the separate beams sweep distinct range swaths. The separate range swaths can be separ-  
15 ated on the basis of the Doppler frequencies of the returned signal, either in the frequency plane of the radar processor, or in the image plane, as will be described more fully with reference to FIG. 2b. Each range swath image for the separate beams is processed separately.

20 Although in the example of global oceanography referred to hereinbefore, the range swaths are preferably 10 km wide and centered at 100 km intervals up to 1500 km, FIG. 1 illustrates only three range swaths centered at 10 km. This is only for simplicity in the illustration. In  
25 practice the range swaths would be separated by greater range intervals. As the SAR sends out a succession of chirped pulses, the time delay for the pulse return provides the range resolution for the images, and Doppler processing of the returns for a series of pulses provides azimuth reso-  
30 lution.

While the time between successive pulses determines the maximum range that a synthetic aperture radar can map, too low a pulse rate can not be relied upon to extend range because it may result in azimuth ambiguities due to Doppler foldover of the matched filtered image output, as pointed out hereinbefore. This multibeam technique allows relatively high pulse repetition rates to be used to avoid that problem, while still extending the range by using different beams at distinct azimuth angles for different range swaths, as illustrated in FIG. 1. The beam parameters are adjusted so that the return from each successive beam is received during successive interpulse periods of the radar system. The Doppler frequency spectrum for the return from each beam is proportional to the azimuth angle of the beam. Therefore, in the processing of the signal, the return from each beam may be separated on the basis of its Doppler frequency spectrum at the frequency plane of the processor in a range telescope 16 comprised of lens  $L_1$  and lens  $L_2$  and an appropriate aperture A, as shown in FIG. 2a. Alternatively, the image formed by each beam can be spatially separated in the azimuth direction by an azimuth telescope  $L_3$  and an appropriate slit S in the recording plane of the processor can be used to record the image from individual beams, as shown in FIG. 2b.

In the design of the system, the interpulse time is selected to be the time for the radar signal to travel across each range strip. The individual beams are chosen, however, such that the sum of the azimuth angles spanned by all the beams is not greater than the maximum angle permitted for a single beam radar for the same pulse repetition fre-

quency. Thus, in the processing of the radar data, the Doppler returns of the individual beams do not interfere with the Doppler foldover due to the discreteness of the azimuth modulation.

5 In deriving the properties of the signal and the image formed by this multiple-beam system we follow the approach described in "Focusing Effects in the Synthetic Aperture Radar Imaging of Ocean Waves," Appl. Phys., Vol. 15, pp. 323-333, 1978 by the present inventor.  $P(x, R, t)$  is  
10 the scattering cross-section of the surface,  $x$  the azimuth coordinate,  $R$  the range,  $t$  the time co-ordinate,  $v_a$  the aircraft is at the coordinate  $x_a$  also equal to  $v_a t_a$ , and  $R_t$  the distance between the radar and some point on the surface at position  $(x, R)$ . At each successive position of the aircraft,  
15 the radar sends out a pulse of the form  $E_R \exp [-i(2\pi f_x t - \alpha t^2)] \text{rect}(t/\tau)$  where  $E_R$  is the amplitude of the signal,  $f_x$  the radar frequency,  $\alpha$  the rate of frequency change and  $\tau$  the duration of the pulse. The illumination field pattern on the terrain at time  $t_a$ , for the multiple beam system, is  
20 described by the aperture function

$$\sum_j A_j \left( \frac{x - x_j - t_a v_a}{\Delta x_j}, \frac{R - R_j}{\Delta R_j} \right) \quad (1)$$

$x_j + t_a v_a$  and  $R_j$  are the azimuth and range coordinates of the  $j^{\text{th}}$  beam  $A_j \left( \frac{x - x_j - t_a v_a}{\Delta x_j}, \frac{R - R_j}{\Delta R_j} \right)$  is the function

describing the pattern of illumination for this beam and  
25  $\Delta x_j$ ,  $\Delta R_j$  are the widths of this function in the azimuth and range dimensions respectively. The amplitude of the signal detected by the radar receiver is proportional to

$$\sum_j \iint \sigma_j \rho(x, R, t) A_j \left( \frac{x - x_j - t_a v_a}{\Delta x_j}, \frac{R - R_j}{\Delta R_j} \right) \exp \left[ i \left( 2\pi f_r \left( t - \frac{2}{c} R_t \right) - \alpha \left( t - \frac{2}{c} R_t \right)^2 \right) \right] \text{rect} \left( \frac{t - \frac{2}{c} R_t}{\tau} \right) dR dx. \quad (2)$$

$\sigma_j$  is a constant depending upon the antenna gain, amplitude  $E_R$  of the radiated pulse, and the range  $R$  to the illuminated area. We approximate  $R_t$  by

$R + \frac{(x_a - x)^2}{R}$ , neglect the  $\frac{(x_a - x)^2}{2R}$  quantity in the non-phase terms and neglect  $\frac{(x_a - x)^2}{Rc}$  in the chirp frequency modulation

function as described in Applied Physics, supra.

The return from each successive beam is recorded at a separate interpulse time where the time coordinate  $t_a$ , equal to  $\frac{x_a}{v_a}$ , describes in the time coordinate frame the aircraft position during which this return is received. The time coordinate covering the duration of this interpulse time is  $t$ . During each  $(t_a, t)$  coordinate on the signal return, the field detected contains contributions from all pulses radiated at times  $(t_a - \frac{2R_j}{c})$  and the return from each pulse, during an interpulse time, is recorded at  $(t + \frac{2R_j}{c})$ . Thus, at any given aircraft position  $x_a$ , at some time  $t_a$ , the signal recorded as a function of  $t$  is given by

$$e_s(t_a, t) = \sum_j \iint \sigma_j \rho(x, R, t_a - \frac{2R_j}{c}) A_j \left( \frac{x - x_j - (t_a - \frac{2R_j}{c}) v_a}{\Delta x_j}, \frac{R - R_j}{\Delta R_j} \right) \exp \left[ i \left\{ \phi_j + \beta \left( t_a - \frac{2R_j}{c} - \frac{x}{v_a} \right)^2 + \alpha \left( t - \frac{2}{c} (R - R_j) \right)^2 \right\} \right] \text{rect} \left( \frac{t - \frac{2}{c} (R - R_j)}{\tau} \right) dx dR \quad (3)$$

where

$$\phi_j = \frac{4\pi (R - R_j)}{\lambda_r}, \quad \beta = \frac{2\pi v_a^2}{\lambda_r R}$$

The quantities  $\phi_j, \beta$  arise from expanding the term  
 $\exp \left[ -i(2\pi f_r(t - \frac{2R_j}{c})) \right]$  where the substitutions  $(t + \frac{2R_j}{c},$   
 $t_a - \frac{2R_j}{c})$  have been used for  $(t, t_a)^C$  as returns from the  $j^{\text{th}}$   
beam at times  $(t, t_a)$ . The term  $\exp [-i2\pi f_r t]$  has been  
5 assumed to have been removed by the demodulation process at  
the receiver. The returns received at separate interpulse  
times are recorded adjacent to each other in the signal,  $t_a$   
being the horizontal coordinate for the film recording and  
 $t$  the vertical coordinate. Since  $t_a$  is discrete, the signal  
10 recorded by the radar receiver can be described by

$\{e_s(t_a, t) \sum_{n=-\infty}^{\infty} \delta(t_a - n\Delta t_p)\}$  where  $\Delta t_p$  is the total time  
duration between adjacent recording times and is generally  
assumed equal to the total interpulse time for the radar.  
In the optical matched filtering of the radar signal, the  
15 spectrum may be displayed at the frequency plane and the  
Fourier transform of (1), denoted by  $U(f_d, f_n)$ , for a  
stationary surface is given by the integral

$$\iint_{-\infty}^{\infty} \left\{ e_s(t_a, t) \sum_{n=-\infty}^{\infty} \delta(t_a - n\Delta t_p) \right\} \exp \left[ -i2\pi(f_d t_a + f_n t) \right] dt_a dt. \quad (4)$$

In evaluating  $U(f_d, f_n)$  for a stationary surface,  
20 i.e.,  $\rho(x, R, t)$  is equal to  $\rho(x, R)$ .  $A_j(\frac{x}{\Delta x_j}, \frac{R}{\Delta R_j})$  is assumed  
to be equal to  $\left[ \text{rect}(\frac{x}{\Delta x_j}) \text{rect}(\frac{R}{\Delta R_j}) \right]$  where  $\text{rect}(x)$  is  
the rectangle function and is equal to unity for  $x$  less  
than half and zero otherwise. The quantities  $(\beta, \alpha, \tau, \Delta t_{aj})$ ,  
where  $\Delta t_{aj}$  is equal to  $\frac{\Delta l_j}{v_a}$ , are assumed large. The transform  
25 of the function  $\left[ \text{rect}(\frac{t}{\delta}) \exp[j\pi\alpha^2] \right]$  is known to be

$$\left[ \left( \frac{1}{\sqrt{|\alpha|}} \right) \text{rect} \left( \frac{f}{\alpha \delta} \right) \exp \left[ j \left\{ \frac{\pi f^2}{\alpha} + \frac{\pi}{4} \text{sgn}(\alpha) \right\} \right] \right],$$

this result having been evaluated using the method of stationary phase, and where  $\alpha, \delta$  have been assumed large numbers. By using substitution of variables, and this

5 result, the transform of the radar signal is found to be

$$U(f_d, f_n) = \sum_j \sum_p \int_{-\infty}^{\infty} \frac{\pi \sigma_j \Delta t_p}{|\alpha \beta|^{\frac{1}{2}}} \rho(x, R) \text{rect} \left( \frac{R - R_j}{\Delta R_j} \right) \text{rect} \left( \frac{\pi f_n}{\alpha} \right) \text{rect} \left( \frac{\pi(f_d + f_{dj} - \frac{n}{\Delta t_p})}{\beta \Delta t_{aj}} \right) \\ \exp \left[ -i\pi^2 \left\{ \frac{(f_d + f_{dj} - \frac{n}{\Delta t_p})^2}{\beta} + \frac{f_n^2}{\alpha} \right\} \right] \exp \left[ i \frac{\pi}{4} \left\{ \text{sgn}(\beta/\pi) + \text{sgn}(\frac{\alpha}{\pi}) \right\} \right] \\ \exp \left[ i \left\{ \phi_j + \beta \left( \frac{x_j}{v_a} \right)^2 - 2\pi \left[ f_d \left( \frac{2R_j}{c} + \frac{(x - x_j)}{v_a} \right) + 2f_n \left( \frac{R - R_j}{c} \right) \right] \right\} \right] dx dR \quad (5)$$

where  $f_{dj} = \frac{\beta x_j}{\pi v_a}$  and  $\text{sgn}$  denotes the sign function which

10 is positive or negative unity for the argument greater or less than zero, and zero for the argument equal to zero.

We note that, for the return from the  $j^{\text{th}}$  beam of the antenna, the Doppler frequencies have a bandwidth  $\frac{\beta \Delta_j}{\pi v_a}$  centered at  $\frac{\beta x_j}{\pi v_a}$ . The return from each beam may therefore

15 be separated at the frequency plane of the optical processor and the unwanted returns blocked off. Provided the processor parameter  $\beta$  includes the range  $R_j$ , of the selected beam, and the interpulse time  $t_a$  chosen so that there is no overlap between the Doppler foldover and the Doppler spectrum for  
20 the individual beams, the image for each beam may be recorded by successively moving the frequency plane aperture in the Doppler frequency dimension.

In deriving the image of the terrain from the signal, the processing consists of convolving a section of the signal  
25 data with the function  $\exp -i(\beta t_a^2 + \alpha t^2)$ . This yields the

image,  $e_i(t_{ao}, t_o)$  to be equal to  $\int_{-\infty}^{\infty} \{e_s(t_a, t) \text{ rect}(t_a/\Delta t_a)\}$

$$\sum_n \int_{-\infty}^{\infty} \delta(t_a - n\Delta t_p) \exp -i\{\beta(t_{ao} - t_a)^2 + \alpha(t_o - t)^2\} dt_a dt.$$

The function  $\text{rect}(t_a/\Delta t_a)$ , where  $\Delta t_a$  is equal to  $\Delta t_{aj}$ ,

all  $\Delta t_{aj}$  being equal, is included to account for the

5 finite length of the signal being processed at any given instant. This integral may be evaluated by a straightforward substitution of variables and the field at a particular moment in the image plane of the processor is given by

$$e_i(t_{ao}, t_o) = \sum_n \sum_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\tau \Delta t'_a \Delta t_p) \rho(x, R) \text{rect}\left(\frac{R-R_j}{\Delta R_j}\right) \text{sinc}\left\{\frac{\Delta t'_a}{\pi} \beta \left(t_{ao} - \frac{2R_j}{c} - \frac{x}{v_a} - \frac{n\pi}{8\Delta t_p}\right)\right\} \text{sinc}\left\{\frac{\alpha\tau}{\pi} \left(t - 2\frac{(R-R_j)}{c}\right)\right\} \exp\left[-i\left\{\beta\left(t_{ao} - \frac{x-x_j}{v_a} - \frac{2R_j}{c}\right)^2 - \beta\left(\frac{x_j}{v_a}\right)^2 + \alpha\left(t_o - \frac{2(R-R_j)}{c}\right)^2 + \frac{2n\pi}{\Delta t_p} \left(\frac{2R_j}{c} + \frac{x-x_j}{v_a}\right)\right\}\right] dx dR \quad (6)$$

if  $\Delta t_a > 0$ , and 0 otherwise,

where

$$\text{sinc}(x) = \frac{\sin \pi x}{\pi x}, \quad \Delta t'_a = \Delta t_a - 2\left(\frac{2R_j}{c} + \frac{(x-x_j)}{v_a}\right),$$

15 We note that, while the image from each beam is separated in the azimuth direction corresponding to the azimuth coordinate of the area illuminated, the range coordinates of all the images span the same range interval, independent of the range interval of the area illuminated. Thus, while  
20 it is not possible to separate the images for different ranges, it is possible to separate them in the azimuth dimension.

In summary, we have calculated in Equation (3) the signal that would be recorded by a multibeam synthetic  
25 aperture radar system, where each beam illuminates a separate range and azimuth footprint such that returns from



each of the beams for a given pulse arrives at separate  
interpulse periods. In Equation (5) we calculate the spectrum  
of this signal and show that the return for a single beam may  
be separated using an aperture in the frequency plane of the  
processor, to only allow transmission of the Doppler frequencies  
corresponding to the selected beam. In Equation (6) we cal-  
culate the image formed at the output of the processor and  
show that the images from the different beams are separated  
in the azimuth dimension, but they all image in the same  
range interval. Thus, images from each beam may be recorded  
individually by using an aperture movable in the azimuth  
direction either in the frequency plane or the image plane  
of the processor, and the separate images combined to provide  
the large swath desired. In the interpretation of this  
analysis, however, we have assumed that the azimuth width  
of each beam is small enough so that the Doppler spectrum  
of the individual beam does not interfere with the Doppler  
foldover due to the finite pulse repetition frequency.

Although particular embodiments of the invention  
have been described and illustrated herein, it is recognized  
that modifications and variations may readily occur to those  
skilled in the art and consequently, it is intended that the  
claims be interpreted to cover such modifications and equi-  
valents.